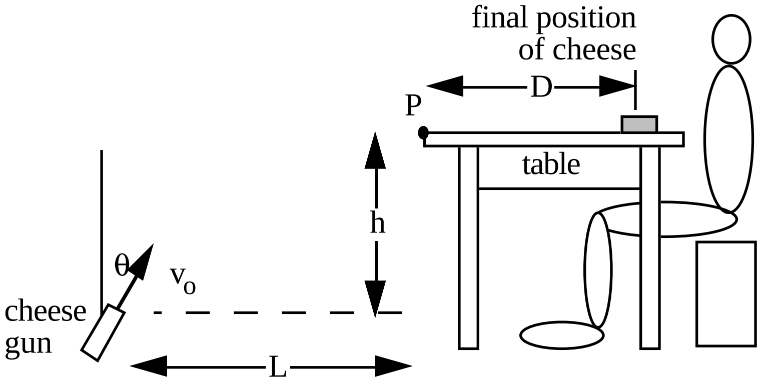
2002 test 1, PHYS 1989-1999

Question 1 (22 marks)



Eccentric inventor Gromit has built a gun that fires pieces of cheese (mass m) at **initial speed v_o,** in the direction the gun is pointed. The end of the gun is a distance h below the top of a table, as shown.

a) The cheese is soft and Gromit wishes it to land it on the table top gently. Derive an expression for the angle θ (with respect to vertical) at which he should he point the gun, so that it reaches the height h with a zero vertical component of velocity. (You may neglect the size of the cheese.)

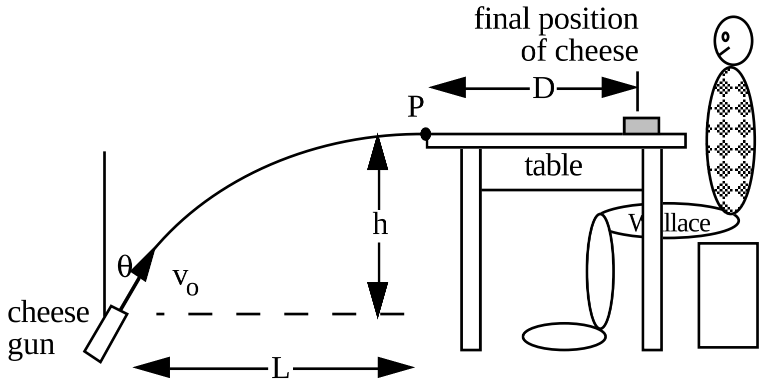
b) The cheese gun is pointed at this angle and Gromit has shaped the cheese so that air resistance is negligible. Derive an expression for the velocity with which the cheese lands on the table, in terms of θ. (Note: give the velocity, not speed).

c) Derive an expression for the distance L from the edge of the table at which he should place the gun so that the cheese lands at point P. Express your answer in terms of θ: you need not relate this to the original parameters of the problem, or try to simplify your answer.

d) The gun is so placed, the cheese lands at point P with zero vertical component of velocity. The cheese then slides a distance D across the table. Determine the work W done by friction in stopping the cheese. Give your answer in terms of v_o and θ.

e) Using the result to (d) or otherwise, determine the coefficient of kinetic friction μ_k between the cheese and the table top that is required to stop the cheese after it has slid the distance D. Give your answer in terms of v_o and θ.

Question 1



a) During flight, the acceleration is –g upwards. The desired v_y is zero, so

$$0 = v_y^2 = v_{yo}^2 + 2a_y(\Delta y) = v_o^2 \cos^2\theta - 2gh$$

$$\therefore \quad v_o^2 \cos^2\theta = 2gh$$

$$\therefore \quad \cos\theta = \frac{\sqrt{2gh}}{v_o}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2gh}}{v_o}\right)$$

b) During flight, no horizontal forces act on the cheese, so v_x is unchanged at $v_o \sin\theta$.
Velocity is $v_o \sin\theta$ in the horizontal direction.

c) Travelling with constant horizontal component of velocity for a time of flight t,
$L = v_o t \sin\theta$.
For the vertical component of displacement
$v_y = v_{yo}t + a_y t$

$$\therefore \quad t = \frac{v_o \cos\theta}{g}$$

$$L = v_o \sin\theta \frac{v_o \cos\theta}{g} \qquad \left(\textit{<optional>} = \frac{v_o^2}{2g} \sin 2\theta \right)$$

d) Horizontal slide, so no change in gravitational potential energy. Therefore the change in kinetic energy $K_f - K_i = W$, the work done by friction.

$$W = 0 - \frac{1}{2} m v_i^2 = -\frac{1}{2} m v_o^2 \sin^2\theta$$

e) During the slide, there is no vertical acceleration so the normal force N = the weight = mg. The friction acts to the left (ie negative in normal convention), so .

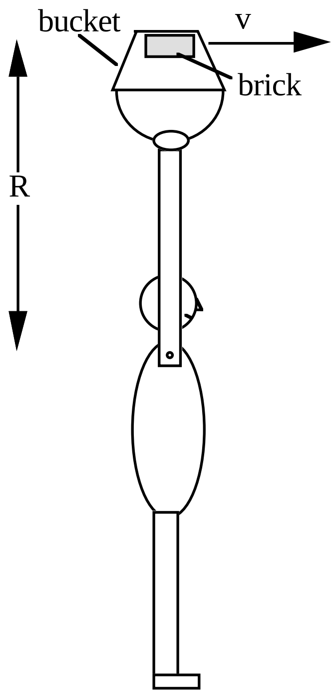
$$F_f = -\mu_k N = -\mu_k mg$$

$$W = F_f D \quad \text{so}$$

$$-\frac{1}{2} m v_o^2 \sin^2\theta = -\mu_k mgD$$

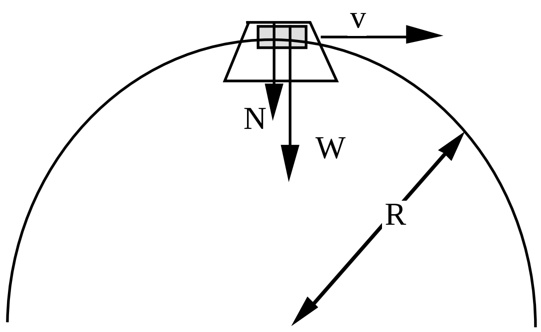
$$\mu_k = \frac{v_o^2 \sin^2\theta}{2gD}$$

Question 2 (8 marks)



A physics lecturer swings a bucket in a vertical circle, radius R, about his shoulder, as shown. The bucket contains a brick. Determine the minimum speed v that the bucket must have at the top of the circle so that the brick stays in contact with the bucket?

Question 2 (marks)



If the brick is in contact with the bucket, then both are travelling in a circle with speed v. The centripetal acceleration is

$$a_c = \frac{v^2}{R} \quad \text{down.}$$

Newton's second law for the vertical direction gives

$$N + W = ma_c = \frac{mv^2}{R}$$

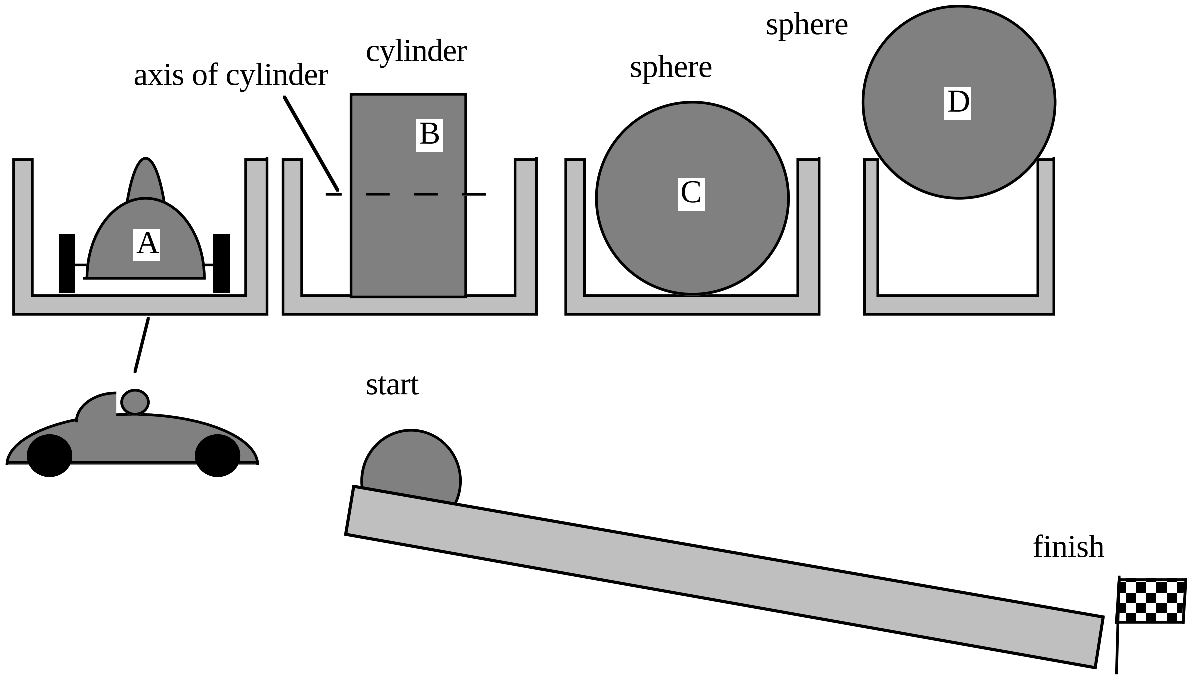
To remain in contact, $N \geq 0$ so

$$\frac{v^2}{R} - W \geq 0 \quad \text{so} \quad v^2 \geq gR$$

so $|v| \geq \sqrt{gR}$

Question 3 (13 marks)

The Australian Grand Prix has been cancelled. You decide to offer an alternative event.



The contestants are two identical brass spheres, a brass cylinder (whose axis is horizontal so it can roll), and a toy racing car. All have the same mass. The wheels of the car are light and they turn with negligible friction on the axle. They roll down four tracks, which are shown in cross section in the top sketch. The tracks are straight, but inclined downwards (all at the same angle). Track A is narrower than the sphere, as shown. The friction between the track and the objects is sufficiently high that the sphere, cylinder and wheels all roll. Air resistance and other losses are negligible.

They race in pairs, and are released from rest at the same time.

You may use without proof $I_{sphere} = \frac{2}{5} mR^2$ and $I_{cylinder} = \frac{1}{2} mR^2$

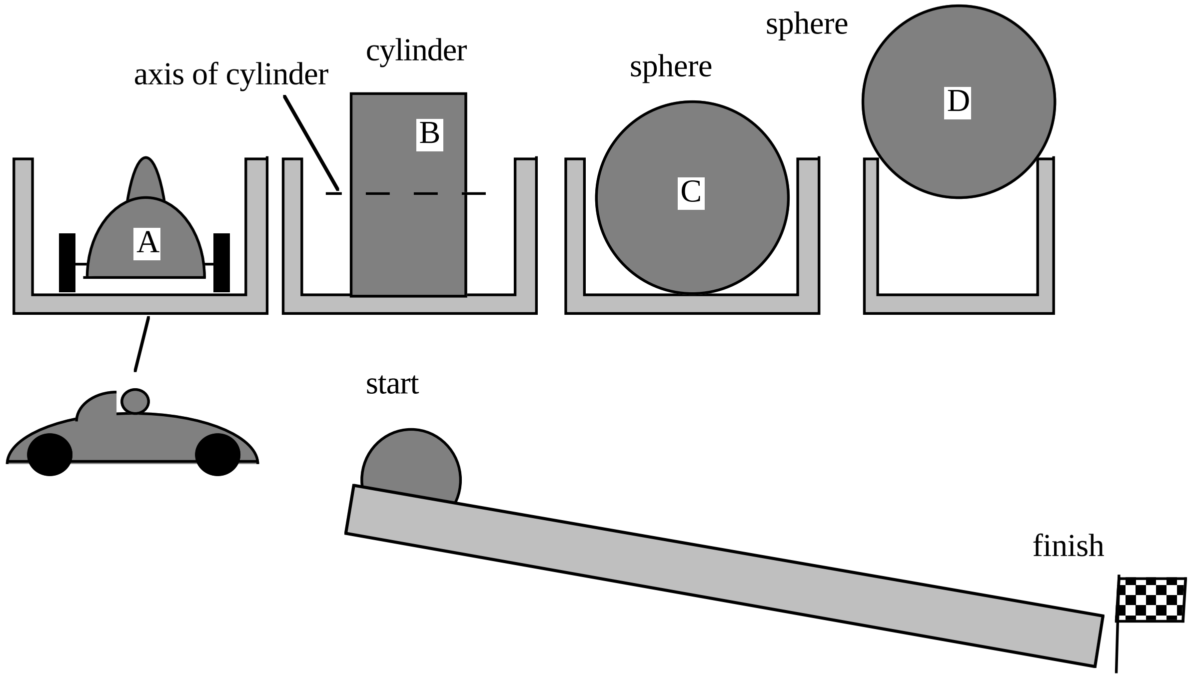
i) In the first heat, A and B race. Which will win? Explain your answer. (You may use equations if you like, but this is not required. A few clear sentences could be enough.)

Hint: it may be helpful to state some general principles that will be relevant to all of (i), (ii) and (iii).

ii) In the second heat, B and C race. Which will win? Explain your answer. (Here you probably will need an equation or two, plus some explanation.)

iii) In the third heat, C and D race. Which will win? Explain your answer. (You may use equations if you like, but this is not required. A few clear sentences could be enough.)

Question 3 (marks)



In all cases friction acts, but they roll, so there is no relative velocity at the point of contact, so friction does no work. In all cases, they convert the *same* initial amount of gravitational potential energy U_g into kinetic energy. Their kinetic energy includes translational kinetic energy ($K_t = mv^2/2$) and rotational kinetic energy ($K_r = I\omega^2/2$).

i) The wheels of the car have negligible mass and therefore negligible rotational kinetic energy, so all of the U_g is turned into K_t. The cylinder converts the same U_g into both K_t and K_r, so its K_t must be smaller. The car wins.

ii) Initial mechanical energy = final mechanical energy

$$mgh = m\frac{v^2}{2} + I\frac{\omega^2}{2}$$

Rolling on edge, $\therefore$ $v = R\omega$, so

$$mgh = m\frac{v^2}{2} + I\frac{v^2}{2R^2}$$

$I_{sphere} < I_{cylinder}$ $\therefore$ $|v_{sphere}| > |v_{cylinder}|$. Sphere wins.

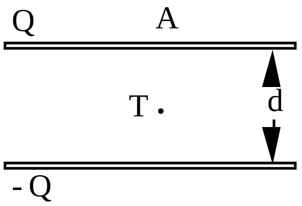
iii) As above, rolling on edge: $mgh = m\frac{v^2}{2} + I\frac{v^2}{2R^2}$

for sphere rolling on $r < R$ $mgh = m\frac{v_D{}^2}{2} + I\frac{v_D{}^2}{2r^2}$ where I is same, C wins

OR, rolling on $r < R$, same ω gives smaller v, so D loses.

Question 4 (15 marks)

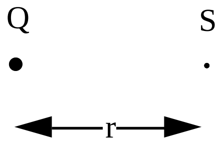
a)



i) Charges Q and – Q are placed on the two plates of a capacitor, with area A and separation d, with no dielectric present. Determine the force on a charge q placed at point T at the middle of the system.

ii) The charge q is removed, and an *uncharged* object is placed at T. Is there a force on the uncharged object and, if so, in which direction is that force? Explain your answer briefly with the aid of a clear, well-labelled diagram.

b)



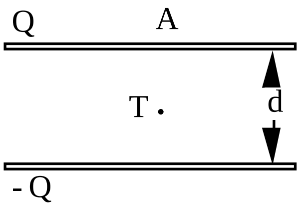
i) Q is the (positive) charge on a small, isolated conductor. What is the force on a charge q placed at point S?

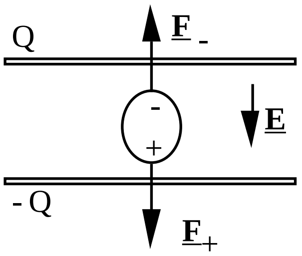
ii) The charge q is removed, and an *uncharged* object is placed at S. Is there a force on the uncharged object and, if so, in which direction is that force? Explain your answer briefly with the aid of a diagram.

iii) With only charge Q and the uncharged object at S present, is there a force on the charge Q? If so, in which direction? Explain your answer in one or two sentences.

Question 4 (marks)

a)





i) **Either:** Field to one plate = $\dfrac{Q}{2\varepsilon_o A}$

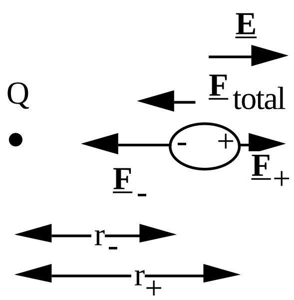
$$\therefore \quad F = Eq = \frac{Qq}{\varepsilon_o A}$$

Or: $E = \dfrac{V}{d} = \dfrac{Q}{Cd} = \dfrac{Qd}{\varepsilon_o Ad}$

$$\therefore \quad F = Eq = \frac{Qq}{\varepsilon_o A}$$

ii) The uncharged object becomes a dipole in the presence of the field **E**. However, the field between the plates of a parallel plate capacitor is ***uniform***, so the field is the same at the positions of the positive and negative charges. The forces on these charges are therefore equal and opposite, as shown, so the total force on the object is zero.

b)

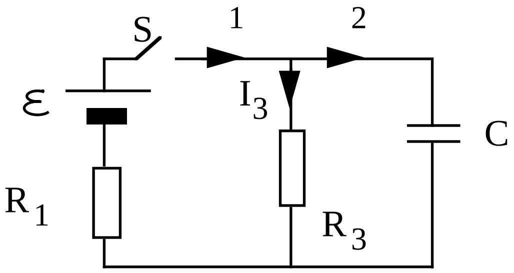


i) Force between two charges $F = \dfrac{kQq}{r^2}$ in the direction of increasing separation, so $\mathbf{F} = \dfrac{kQq}{r^2}$ *to the right*.

ii) The uncharged object becomes a dipole in the presence of the field **E**. The field near an isolated is ***non uniform***, being greater closer to the charge, so the force on the positive charge is less than that on the negative charge. So the net force is to the left, as shown.

iii) The charge Q exerts a force on the uncharged object, so by Newton's third law, the uncharged object exerts an equal and opposite force on the charge Q. This force is to the right.

Question 5 (13 marks)



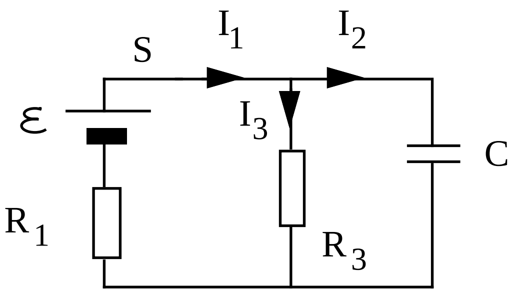
In the circuit shown at left, the capacitor C is initially uncharged and the switch S is open.

a) Give expressions for the currents I_1, I_2 and I_3, at a time immediately after the closing of the switch S. Explain briefly how you arrived at your answers.

Hint: this does not require differential equations, and it may be helpful to consider the state of C.

b) Give expressions for the currents I_1, I_2 and I_3, at a time long after the closing of the switch S. Explain briefly how you arrived at your answers.

Question 5



a) Immediately after the switch is closed, the capacitor is still uncharged, so the voltage across C is zero. Therefore the voltage across R_3 is also zero, so

$$I_3 = 0.$$

Therefore, by Kirchoff's current law, $I_1 = I_2$. Applying Kirchoff's loop law,

$$\mathcal{E} = I_1R_1 + V_C = I_1R_1 + 0$$

$$I_1 = \mathcal{E}/R_1 = I_2.$$

b) A long time after S is close, the capacitor is no longer charging, so

$I_2 = 0.$

Therefore $I_1, = I_3$. Applying Kirchoff's loop law,

$$\mathcal{E} = I_1R_1 + I_3R_3 = I_1(R_1 + R_3)$$

$$I_1 = I_3 = \frac{\mathcal{E}}{R_1 + R_3}$$