PHYS 1189—1199

Lecturers Joe Wolfe Kristina Wilk

Texts:

- Physics, Kane & Sternheim, (Wiley).

  Thorough text, very well written and illustrated, and could serve as a reference book for years to come.

- The Physics Companion A.C. Fischer-Cripps. (Fischer-Cripps Laboratories). Small, cheap, good text covering most of the syllabus. Available at the CoOp bookstore.

  on-line: www.fclabs.com.au/companions good web text, covers much of the syllabus.

  Not free: A$10 for access for one year.

Your web page

www.phys.unsw.edu.au/~jw/1189.html

Tutorials

Tut sheets have study notes and questions

Physics: "The science treating of the properties of matter and energy, or of the action of the different forms of energy on matter in general (excluding chemistry and biology)" (OED)

Vectors have magnitude and direction

scalars only have magnitude

Examples

The Earth's gravitational field in this room?

$\underline{\mathbf{g}}$ = 9.8 N.kg^{-1} **down** **vector**

The volume of milk in a carton?

600 ml (= 6 10^{-4} m^3) scalar

The spin of the Earth?

one turn per 23.9 hours (axis is) **North** **vector**

= 7.30 10^{-5} rad.s^{-1} **North**

Scalars: mass, length, heat, temperature...

Vectors: Force, acceleration, electric field...

2 m towards door; 3.7 ms^{-1} at 31° E. of N.,

- 9.8 ms^{-2} up, 4 N in +ve x direction

Vectors written bold, or underscored: $\underline{\mathbf{a}}$

---

displacement $\underline{\mathbf{r}}$	distance r
how far did you move *and in what direction?*	how far did you go?
20 cm up	

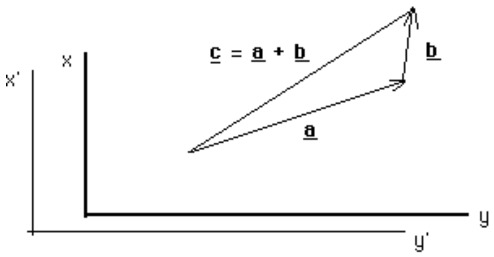
---

velocity $\underline{\mathbf{v}}$	speed v
how fast are you going *and in what direction?*	how fast are you going now?
5 km.hr^{-1} East	

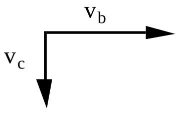
---

v is the magnitude of $\underline{\mathbf{v}}$

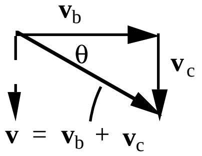
Vector addition



Example



A boat heads East at 8 km.hr^{-1}. The current flows South at 6 km.hr^{-1}. What is the boat's velocity relative to the Earth?



$\mathbf{v} = \mathbf{v}_{boat} + \mathbf{v}_{current}$

To add the vectors, draw them head-to-tail.

magnitude: $v = \sqrt{v_b^2 + v_c^2}$

$= \sqrt{(8 \text{ km.hr}^{-1})^2 + (6 \text{ km.hr}^{-1})^2}$

$= \sqrt{(8^2 + 6^2)\,(\text{km.hr}^{-1})^2}$

$= \sqrt{(64 + 36)}\ \sqrt{(\text{km.hr}^{-1})^2}$

$= 10 \text{ km.hr}^{-1}$

direction: $\theta = \tan^{-1} \dfrac{6 \text{ km.hr}^{-1}}{8 \text{ km.hr}^{-1}} = \tan^{-1} 0.75$

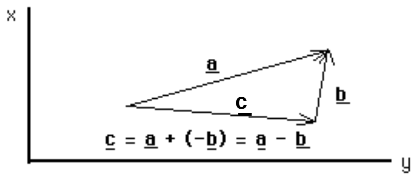
$= 37°$

Answer: 10 km.hr^{-1} at 37° South of East

Vector subtraction

Draw the vectors head to head to subtract them.

Example A sailor wants to travel East at 8 km.hr^{-1}. The current flows South at 6 km.hr^{-1}. What direction must she head, and what speed should she make relative to the water?



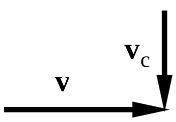
<u>**a**</u> - <u>**b**</u> = <u>**c**</u> i.e. <u>**a**</u> = <u>**c**</u> + <u>**b**</u>

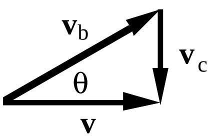
In other words, you can rearrange vector subtraction so that it becomes vector addition

$\mathbf{v} = \mathbf{v}_{boat} + \mathbf{v}_{current}$

$\mathbf{v}_{boat} = \mathbf{v} - \mathbf{v}_{current}$

To subtract the vectors, draw them head-to-head.





magnitude: $v_b = \sqrt{v^2 + v_c^2}$

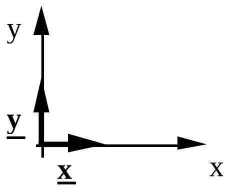
$= \sqrt{(8 \text{ km.hr}^{-1})^2 + (6 \text{ km.hr}^{-1})^2}$

$= 10 \text{ km.hr}^{-1}$

direction: $\theta = \tan^{-1} \dfrac{v_c}{v} = \tan^{-1} \dfrac{6 \text{ km.hr}^{-1}}{8 \text{ km.hr}^{-1}} = 37°$

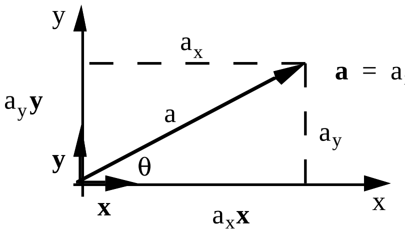
She must head 37° North of East and travel at 10 km.hr^{-1} with respect to the water.

Components and unit vectors



$\underline{\mathbf{x}}$ is **unit vector**: magnitude of 1 in x direction.

Similarly $\underline{\mathbf{y}}$



$\mathbf{a} = a_x\mathbf{x} + a_y\mathbf{y}$

a_x is the **component** of a in the x direction - *scalar*

$a_x = a\cos\theta$, $\quad a_y = a\sin\theta$

$\underline{\mathbf{a}} = a_x\,\underline{\mathbf{x}} + a_y\,\underline{\mathbf{y}}$

Pythagoras: $\quad a = \sqrt{a_x{}^2 + a_y{}^2}$

Angle $\quad \theta = \tan^{-1}\dfrac{a_y}{a_x}$

Addition by components

$\underline{\mathbf{c}} = \underline{\mathbf{a}} + \underline{\mathbf{b}} = (a_x\,\underline{\mathbf{x}} + a_y\,\underline{\mathbf{y}}) + (b_x\,\underline{\mathbf{x}} + b_y\,\underline{\mathbf{y}})$

$c_x\,\underline{\mathbf{x}} + c_y\,\underline{\mathbf{y}} = (a_x + b_x)\,\underline{\mathbf{x}} + (a_y + b_y)\,\underline{\mathbf{y}}$

$c_x = a_x + b_x$ and $c_y = a_y + b_y$

Important:

1 vector eqn in n dimensions $\rightarrow$ n independent algebraic eqns.

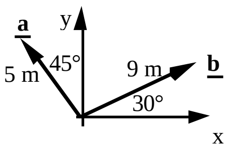
important case: $\quad 0 = 0\,\underline{\mathbf{x}} + 0\,\underline{\mathbf{y}}$

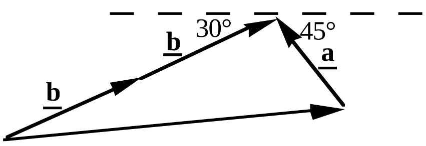
$\therefore \quad$ if $\underline{\mathbf{a}} + \underline{\mathbf{b}} = 0$, $\quad$ (e.g. mechanical equilibrium)

$a_x + b_x = 0$ ***and*** $a_y + b_y = 0$

Example

$\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ are defined in the diagram at right. Calculate $2\underline{\mathbf{b}} - \underline{\mathbf{a}}$





looks messy

Alternatively:

$\underline{\mathbf{b}} = 9\cos 30°\,\underline{\mathbf{x}} + 9\sin 45°\,\underline{\mathbf{y}}$ m $\quad$ and

$\underline{\mathbf{a}} = -5\cos 45°\,\underline{\mathbf{x}} + 5\sin 45°\,\underline{\mathbf{y}}$ m

$$2\underline{\mathbf{b}} - \underline{\mathbf{a}} = (18\cos 30° + 5\cos 45°)\,\underline{\mathbf{x}} + (18\sin 30° - 5\sin 45°)\,\underline{\mathbf{y}} \text{ m}$$

$$= 19\,\underline{\mathbf{x}} + 5.5\,\underline{\mathbf{y}} \text{ m}$$

$$|2\underline{\mathbf{b}} - \underline{\mathbf{a}}| = 20 \text{ m}$$

$$\theta = \tan^{-1}\frac{5}{19} = 16°$$

Kinematics - study of motion

Motion with constant acceleration in y dir^n

$a_y \quad = \text{ constant } \equiv \dfrac{dv_y}{dt}$

$v_y \quad = \int a_y \, dt$

$\quad\quad = a_y t + \text{const}$

$v_y \quad = u_y + a_y t \qquad\qquad\qquad \text{(i)}$

$y \quad = \int v_y \, dt$

$\quad\quad = \int u_y + a_y t \; dt$

$\quad\quad = u_y t + \frac{1}{2} a_y t^2 + \text{constant}$

$y \quad = y_o + u_y t + \frac{1}{2} a_y t^2. \qquad\qquad \text{(ii)}$

Eliminate t from (i) and (ii)

$y - y_o \quad = u_y t + \frac{1}{2} a_y t^2 \;\; = \; .... \Rightarrow$

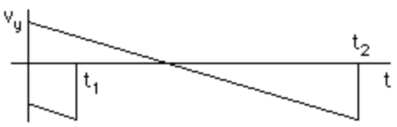
$2a_y(y - y_o) = v_y^2 - u_y^2 \qquad\qquad \text{(iii)}$

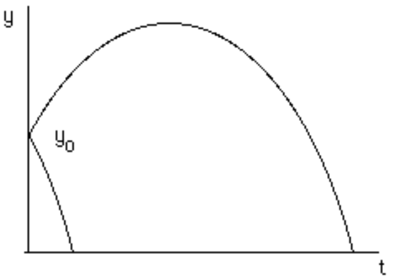
symbols are not always the same

Example. Ball 1 thrown vertically up at 5 ms^{-1} from 20 m above ground. Simultaneously, ball 2 thrown vertically down at 5 ms^{-1} from 20 m above ground. What are their speeds when they hit the ground, and the interval between collisions?

u_{y1} = 5 ms^{-1} $\qquad$ u_{y2} = - 5 ms^{-1} $\qquad$ y_o = 20 m

Displacement-time and velocity-time graphs are often useful





Example. Ball 1 thrown vertically up at 5 ms^{-1} from 20 m above ground. Simultaneously, ball 2 thrown vertically down at 5 ms^{-1} from 20 m above ground. What are their speeds when they hit the ground, and the interval between collisions?

u_{y1} = 5 ms^{-1} $\qquad$ u_{y2} = - 5 ms^{-1} $\qquad$ y_o = 20 m

When y = 0, t = ? v = ?

Strategy:

$y \; = \; y_o + u_y t + \frac{1}{2} a_y t^2 \quad \rightarrow \quad t_1 \text{ and } t_2.$

$v_y \; = \; u_y + a_y t \quad \rightarrow \quad v_{y1} \text{ and } v_{y2}.$

$y \;=\; y_o + u_y t + \frac{1}{2} a_y t^2$

$0 \;=\; 20 + 5\,t - 9.8\,t^2$

solve quadratic $\rightarrow$ gives t $\qquad$ (*you know this*)

substitute in $\; v_y \;=\; u_y + a_y t$

Example. Joe runs (v_J = 6 ms^{-1}) towards a stationary bus. When he is 20 m from it, the bus accelerates away at 2 $m.s^{-2}$. Can he overtake it?

Given: $v_J = 6\ ms^{-1}$, $u_b = 0$, $a_b = 2\ m.s^{-2}$, $a_J = 0$,
$x_{bo} - x_{Jo} = 30$ m

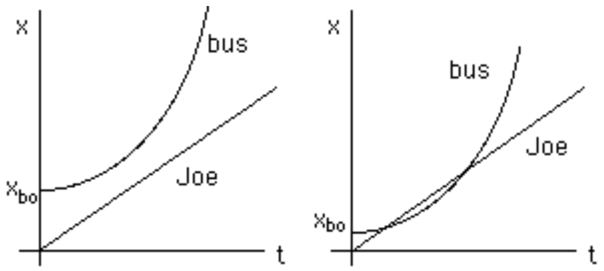
Asked: Does $x_J = x_b$ at any t?

$$x_b = x_{bo} + u_b t + \frac{1}{2} a_b t^2 \qquad \text{(i)}$$

$$x_J = x_{Jo} + u_J t + \frac{1}{2} a_J t^2 \qquad \text{(ii)}$$

Eliminate x_{Jo} by choice of origin, x_{bo} = 30 m

Draw a diagram



'catch' $\rightarrow$ $x_b = x_J$ substitute (i) and (ii)

$$\frac{1}{2} a_b t^2 - u_J t + x_{bo} = 0$$

$$t = \frac{u_J \pm \sqrt{u^2{}_J - 2a_b x_{bo}}}{a_b}$$

$\sqrt{-ve}$ $\therefore$ no solutions, $\therefore$ no overtaking.

Projectiles Without air, $a_y = -g$ = constant. $a_x = 0$

(Galileo: independence of horiz. & vert motion)

(ii) $\rightarrow$ $y = y_o + u_y t - \frac{1}{2} g t^2$.

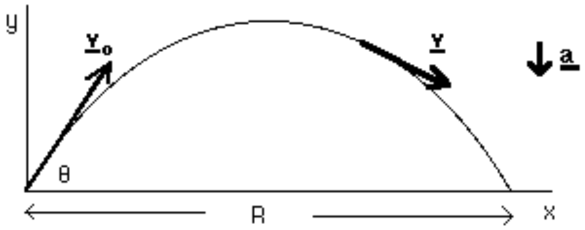
(ii) $\rightarrow$ $x = x_o + v_x t$.

Choose axes so that $x_o = y_o = 0$

Eliminate t using (ii) $t = \frac{x - x_o}{v_x} = \frac{x}{v_x}$

$$\rightarrow \quad y = u_y \left(\frac{x}{v_x}\right) - \frac{1}{2} g \left(\frac{x}{v_x}\right)^2 \qquad (*)$$

which is a parabola. *ie* the **path is a parabola**



We can now solve for the range R.

y = 0 when x = 0 or x = R

$$(*) \Rightarrow \quad u_y \left(\frac{R}{v_x}\right) = \frac{1}{2} g \left(\frac{R}{v_x}\right)^2$$

$$v_{yo} = \frac{1}{2} g \left(\frac{R}{v_x}\right)$$

$$R = \frac{2 v_{yo} v_x}{g}$$

$v_x = u \cos\theta$ $\quad v_{yo} = u \sin\theta$

$$R = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$= \frac{u^2 \sin 2\theta}{g}$$

max of sin function is 1, when $\theta = 45°$

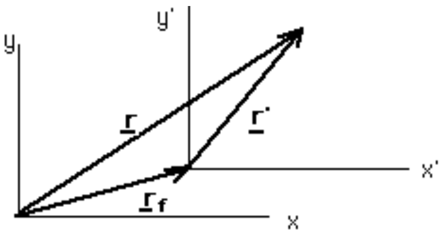
$$R_{max} = \frac{u^2}{g}$$

check units: $[u] = ms^{-1}$, $[g] = ms^{-2}$ $\rightarrow$ $\left(\frac{m}{s}\right)^2 \left(\frac{s^2}{m}\right) = m$

Relative velocities

(Gallilean/Newtonian relativity)

origin of frame (x',y') is at $\underline{\mathbf{r}}_f$ and moves with $\underline{\mathbf{v}}_f$ w.r.t (x,y) frame. No rotation.



$\underline{\mathbf{r}}' = \underline{\mathbf{r}} - \underline{\mathbf{r}}_f$

$$\underline{\mathbf{v}}' = \frac{d}{dt}\underline{\mathbf{r}}'$$

$$= \frac{d}{dt}\underline{\mathbf{r}} - \frac{d}{dt}\underline{\mathbf{r}}_f$$

$$= \underline{\mathbf{v}} - \underline{\mathbf{v}}_f$$

similarly, $\underline{\mathbf{a}}' = \underline{\mathbf{a}} - \underline{\mathbf{a}}_f$

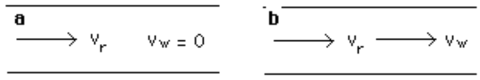
any problems with this argument?

Problem River flows East at 10 km/hr. Sailing boat travels East down the river.

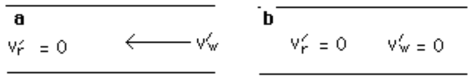
Can the boat travel faster

a) with no wind?

b) with 10 km/hr wind from West?



Consider motion *relative to the water.*



b) $v'_W = 0$ $\therefore$ boat cannot sail

$\therefore$ boat drifts East at 10km/hr.

a) 10 km/hr headwind.

Sailboat can tack

$\therefore$ travel East w.r.t water $\therefore$ goes faster.