PHYS 1199 Test 1, 2003

Question 1. (18 marks)

A scientist is on the roof of a building and wonders how high it is. It is very quiet, and there are no people below, so she drops a bag of water. Counting seconds, she then times the interval T between when she releases the bag and when she hears the sound of the collision between the bag and the ground. She then neglects the time taken for the sound to reach her, and calculates the height of the building.

i) Draw a neat displacement-time graph for the position of the falling bag (you may neglect air resistance). Indicate the height h of the building and the time T_1 taken for the bag to fall to the ground.

ii) Showing your working, relate the height h of the building to T_1 and to other relevant constants.

iii) The building is in fact 80.0 m tall. Take g = 9.80 ms^{-2} and calculate T_1 to 3 significant figures.

iv) On your displacement-time graph, show the displacement of the sound wave pulse that travels from the ground up to the scientist on top of the roof. Your graph need not be to scale.

v) Taking the speed of sound at 344 ms^{-1}, calculate T_2, the time taken for the sound to travel from the collision to reach the scientist on the roof, also to 3 significant figures. Show T_2 on your graph.

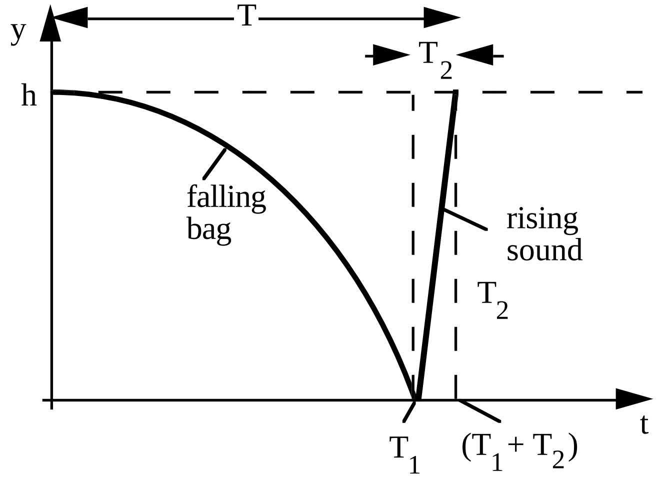
vi) State the time T between release of the bag and arrival of the sound. Think carefully about the number of significant figures

Recall that our scientist (because she is calculating in her head) neglects time taken for the sound signal. Further (also because she is calculating in her head) uses g = 10 ms^{-2}.

vii) What value does the scientist get for the height of the building?

viii) Comment on the accuracy under the circumstances.

Question 1.



ii) $y = y_o + v_{yo}t + \frac{1}{2}a_y t^2$

$= h + 0 - \frac{1}{2}gt^2$

hits the ground when y = 0, so

$0 = h - \frac{1}{2}gT_1^2$

$\therefore \quad T_1^2 = \frac{2h}{g}$

$T_1 = \sqrt{\frac{2h}{g}}$

iii) $h = 80.0$ m $\rightarrow$ $T_1 = 4.04$ s.

v) speed = distance travelled/time taken, so $T_2 = h/v_s = 0.233$ s.

vi) $T = T_1 + T_2 = 4.04 \text{ s} + 0.233 \text{ s} = 4.27$ s.

vii) She says $0 = h - \frac{1}{2}gT_1^2$

$\therefore \quad h = \frac{1}{2}gT_1^2 \cong \frac{1}{2}gT^2.$

So she calculates $h \cong \frac{1}{2}(10\text{ms}^{-2})(4.27\text{s})^2 = 91$ m.

viii) The error of 14% is probably comparable with the error in timing. If she had a stopwatch, she possibly would have pencil, paper and a calculator. (*Any reasonable comment about the accuracy earns a mark.*)

Question 2. (18 marks)

i) Assuming the orbit of the Earth about the sun to be a circle with radius $R = 1.50\ 10^{11}$ m, calculate the magnitude of the Earth's angular acceleration. Neglect the motion of the sun.

ii) State the direction of the angular acceleration in (i).

iii) The constant of Gravitation is $G = 6.67\ 10^{-11}\ Nm^2kg^{-2}$. Use this value and your answer to (i) to determine the mass M of the sun.

iv) The moon has mass m_m 7.36 10^{22} kg. The Earth has mass m = 5.98 01^{24} kg. The sun has a mass M = 1.99 10^{30} kg.

The distance sun-earth = R = 1.50 10^{11} m. The distance earth-moon = r = 3.82 10^8 m.

At new moon, the moon lies on a line between the Earth and the sun and is at a distance r = 3.82 10^8 m from the Earth. Calculate the total gravitational force on the moon due to the sun and the Earth. (Hint: a diagram may be helpful)

v) State the direction of the force in (iv)

vi) State the magnitude of the acceleration of the moon at new moon, due to the forces exerted by the sun and the earth.

vii) State the direction of the acceleration in (vi).

viii) Compare your answers for (i & ii) and (vi & vii) and comment briefly (about two or three sentences).

Question 2.

i) Let T be the period of the Earth's orbit, ie one year.

$$a_c = R\omega^2 = R\left(\frac{2\pi}{T}\right)^2 = 1.50\ 10^{11}\ m\left(\frac{2\pi}{365*24*60}\right)^2 = 5.95\ 10^{-3}\ ms^{-2}$$

ii) towards the sun

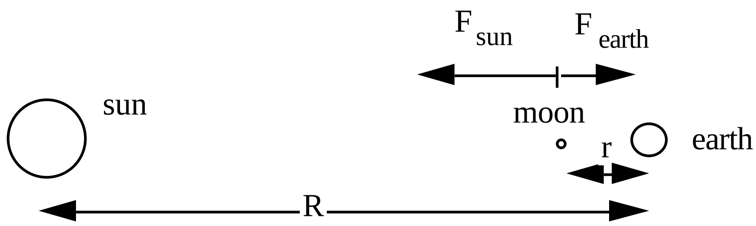
iii) for any body of mass m orbiting the sun in the earth's orbit:

$$ma_c = F_g = \frac{GMm}{R^2}$$

$$a_c = \frac{GM}{R^2}$$

$$M = \frac{R^2 a_c}{G} = 2.01\ 10^{30}\ kg.$$

iv)



$$\Sigma F = F_{sun} - F_{earth} = \frac{GMm_m}{(R-r)^2} - \frac{Gmm_m}{r^2} \cong Gm_m\left(\frac{M}{R^2} - \frac{m}{r^2}\right) = ... = 2.33\ 10^{20}\ N.$$

v) (*$F_{sun} > F_{earth}$, so it is*) towards the sun (*answer only required, not explanation*)

vi) $a = \Sigma F/m_m = 3.17\ 10^{-3}\ ms^{-2}$.

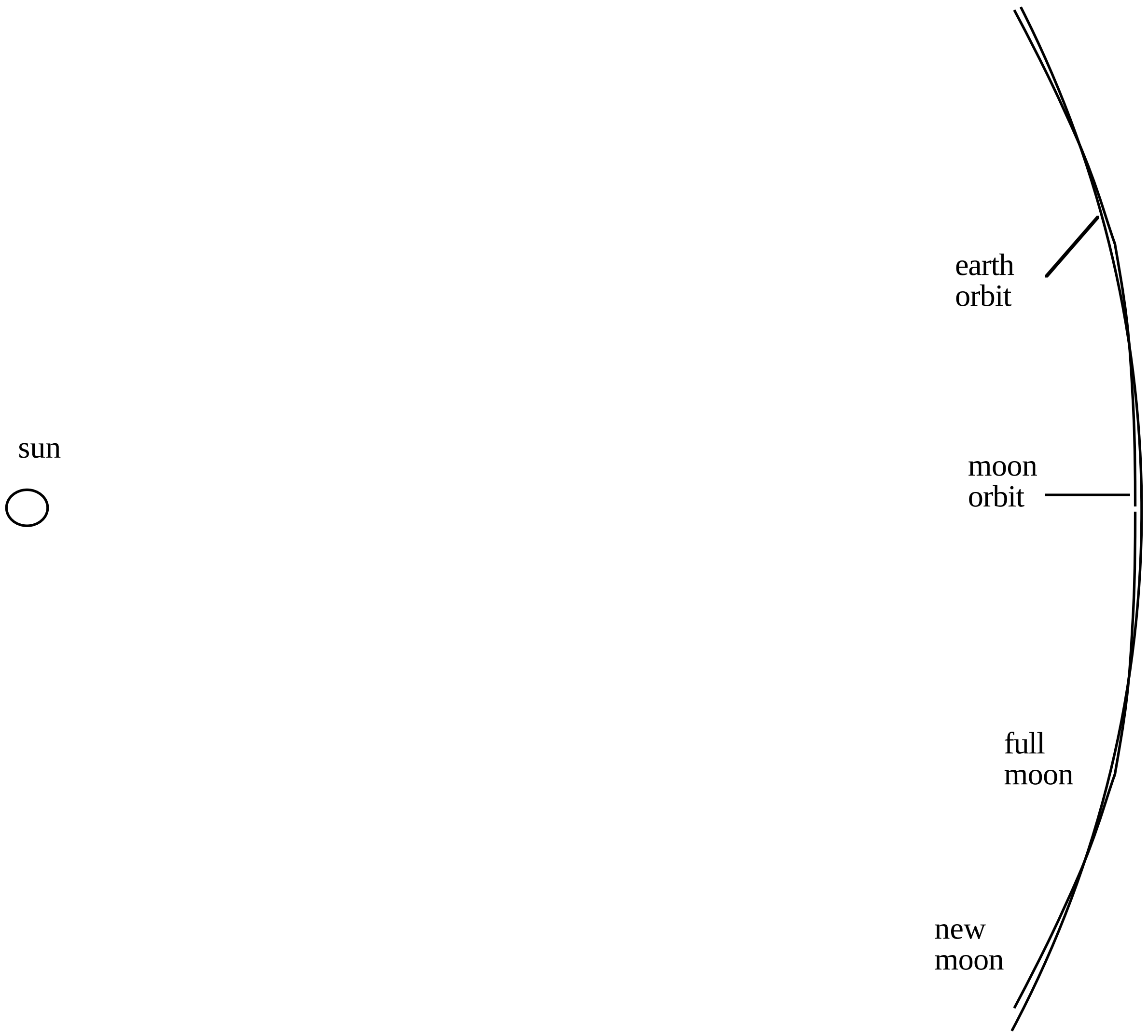
vii) towards the sun

viii) Both earth and moon accelerate towards the sun. At new moon, the moon accelerates at only about half the rate of the earth's acceleration. When it is closer to the sun, it is not accelerating so rapidly towards the sun as the earth is, because it is beginning to move further from the sun.

(*or any other reasoable comments*)

Not for marks: What is interesting, of course, is that at new moon the moon is actually accelerating away from the Earth.

The earth travels in a circular path around the sun, with constant radius R and constant centripital acceleration. The moon's path is not quite circular: it is closer to the sun (R-r) at new moon and further from the sun (R+r) at full moon. Because r << R. Given the disparity, it's actually hard on this scale to show that the moon's orbit is always concave towards the sun, so that's why a diagram was *not* called for in this question.



Question 3. (17 marks)

i) Define the electrical potential difference between two points A and B.

ii) Define equipotential.

iii) What is the direction of the electric field at an equipotential surface?

The figure shows two parallel conducting plates. The upper plate is charged to +3 kV, and the lower plate to –3 kV. The two plates are 18 mm apart.

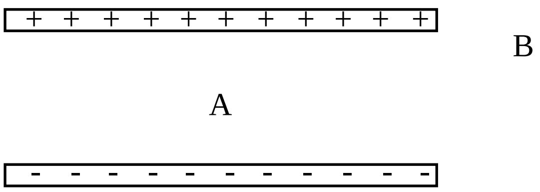
iv) On the figure provided (there is a spare in case you spoil this one), sketch the equipotentials for +2 kV, +1 kV, 0 V, −1 kV and −2 kV. *Your equipotentials should extend at least 20 mm beyond the edges of the plates.*

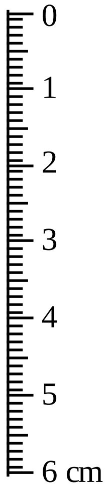
v) Using your answer to (i) or otherwise, *estimate* the magnitude of the electric field at A, and indicate its direction with an arrow on the diagram.

vi) Using your answer to (i) or otherwise, *estimate* the magnitude of the electric field at B, and indicate its direction with an arrow on the diagram. You may use a ruler. If you haven't brought one with you, tear off the ruler provided in the margin. *Only an approximate estimate is required.*

vii) If an uncharged object is placed at A, would it experience an electric force? Briefly explain your answer.

viii) If an uncharged object is placed at B, would it experience an electric force? Briefly explain your answer.





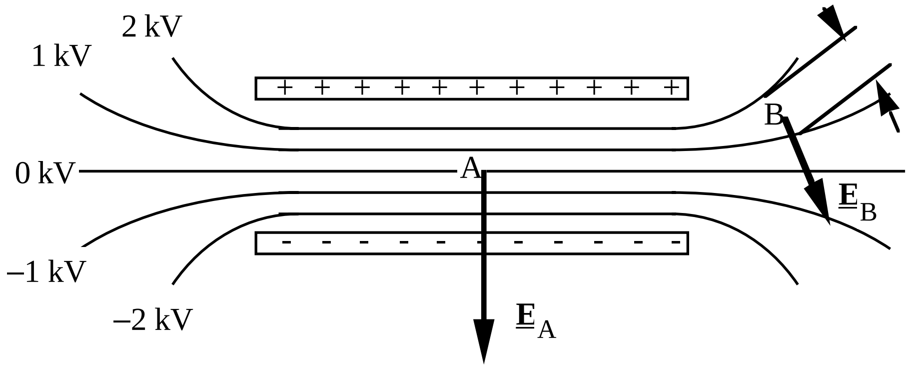
Question 3.

i) The electrical potential V_{AB} between two points A and B is the work done per unit charge to move a test charge q from A to B. ($V_{AB} \equiv W_{AB}/q$.) *Or*

The electrical potential V_{AB} between two points A and B is the difference in the electrical potential energy per unit charge between A and B.

ii) The locus of points having the same electrical potential.

iii) The electric field is perpendicular to an equipotential surface.



iv)

The equipotentials should be equally spaced parallel lines between the plates, where the field is uniform, and they should be further apart outside the plates, where the field is weaker and nonuniform.

v) $V_s \cong -\frac{\Delta V}{\Delta s}$ where s is displacement. At A, $\frac{\Delta V}{\Delta s} \cong \frac{1\text{kV}}{3\text{ mm}} \cong$ 300 kV.m^{-1} or 300 kN.C^{-1}. Direction as shown on the sketch.

vi) At B, the distance between the equipotentials is about 6 mm in my diagram, but might be different in other sketches. This is not important as we only want an approximate estimate.

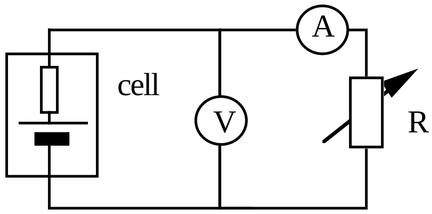
$E_B \cong \frac{1\text{kV}}{6\text{ mm}} \cong$ 200 kV.m^{-1} or 200 kN.C^{-1}. Direction as shown on the sketch.

vii) In an electric field, an uncharged object becomes a dipole. In a uniform field, a dipole is subjected to no force because the equal forces on each charge of the dipole cancel.

viii) In a nonuniform field, a dipole may experience a force, because the two charges of the dipole may be in regions of unequal field.

Question 4. (14 marks)

i)



A set of measurements are undertaken to determine the emf $\mathcal{E}$ and the internal resistance r of a cell. They use an ideal voltmeter, an ideal ammeter and a variable resistance R, as shown. The resistance of the wires is negligible. The measured values are

V	5.5	4.8	3.9	3.0	1.4	0.1	Volts
I	10	25	40	60	92	118	mA

Graphically or otherwise, determine the values of the emf $\mathcal{E}$ and the internal resistance r of the cell.

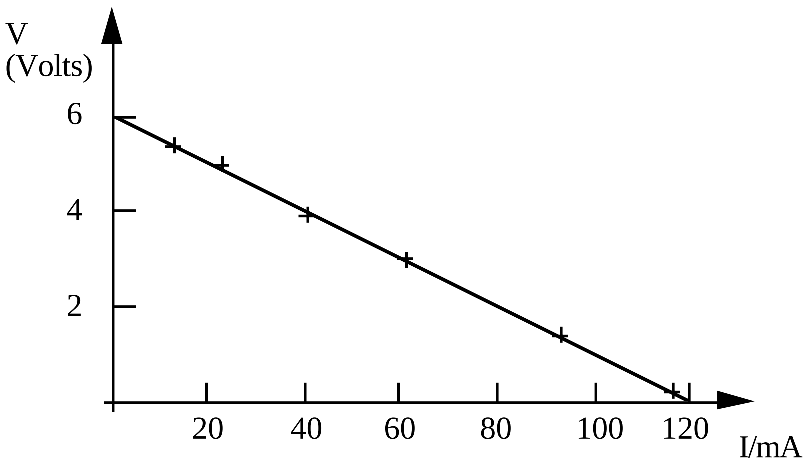
ii) A cell with emf 12.0 V and internal resistance 0.6 Ω is to be charged using an ideal DC source with variable voltage V. The cell is capable of dissipating heat at a rate of 5 W safely, without overheating.

a) Draw a clearly labelled diagram of the circuit and indicate the polarity of the batterty and the direction of current flow.

b) Determine the highest value of V that may safely be used in charging.

Question 4.

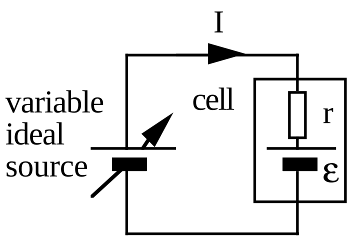
i)



The intercept on the ordinate gives the emf: 6.0 V.

The slope equals minus one times the internal resistance r, which here is 50 Ω.

ii)



The circuit is as shown at left.

The power dissipated as heat is $P_r = I^2 r$

$5\text{ W} = P_{max} = I_{max}^2 r$

$I_{max}^2 = P_{max}/r$

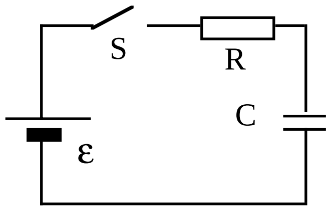
$I_{max} = \sqrt{P_{max}/r} = \ldots = 2.9\text{ A}$

$V_{max} = \varepsilon + I_{max} r = \ldots = 13.7\text{ V.}$

Question 5. (15 marks)

i) From the definition of capacitance, derive an expression for the resistance of a series combination of two capacitors, C_1 and C_2. Show your working.

ii)



In this circuit, the switch is closed at time t = 0, when the capacitor is initially uncharged.

On the axes provided, sketch the following quantities, showing their values for $t < 0$ as well as as for $t > 0$:

a) The voltage V_C across the capacitor. On this sketch, show approximately the characteristic time τ for the circuit, and show the final value of V_R.

b) The voltage V_R across the resistor.

c) The voltage V_ε across the (ideal) emf.

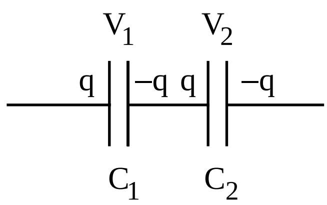
d) The voltage V_S across the switch.

(Hint: check that your sketches satisfy Kirchoff's loop rule)

Remember to show their values for a $t < 0$ as well as for $t > 0$.

Two sets of axes are provided, in case you spoil one

Question 5

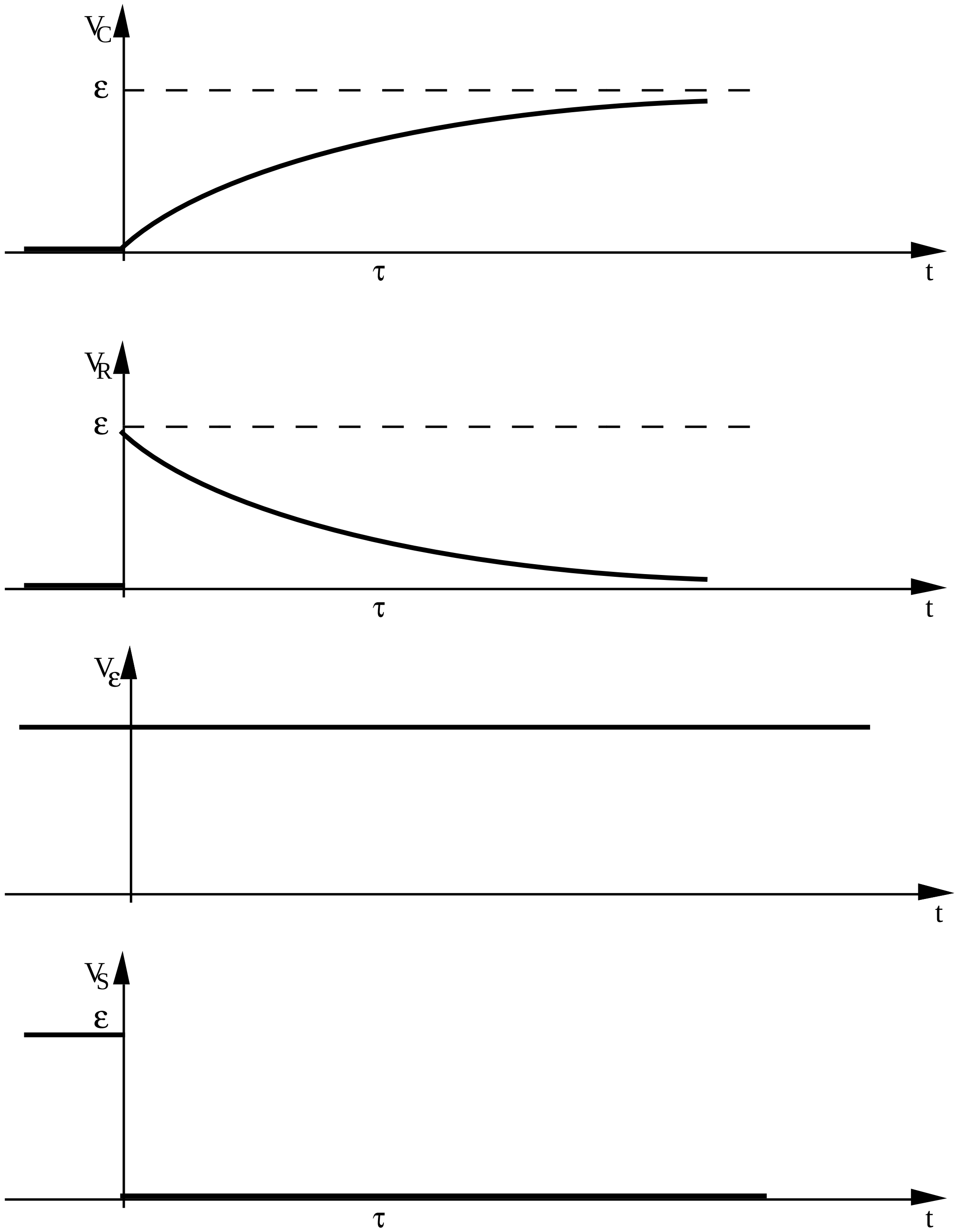


i)

Because the central conductor is not connected to the external circuit, its charge must remain at zero. So the charge on the two capacitors is equal (*this not required for the answer*).

$C_1 \equiv q/V_1$ so $V_1 \equiv q/C_1$ Similarly: $V_2 \equiv q/C_2$

$$C \equiv \frac{q}{V} = \frac{q}{V_1 + V_2} = \frac{q}{q/C_1 + q/C_2} = \frac{1}{1/C_1 + 1/C_2}$$



V_C
ε
τ
t
V_R
ε
τ
t
$V_ε$
t
V_S
ε
τ
t